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HARMONIC-BALANCE ANALYSIS OF MULTITONE AUTONOMOUS NONLINEAR MICROWAVE CIRCUITS

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ABSTRACT

The paper introduces a Newton-iteration based algorithm for the harmonic-balance analysis of autonomous microwave circuits operating in quasi-periodic conditions. The autonomy concept for quasi-periodic regimes is first discussed in detail, and a solution strategy of the harmonic-balance equations based on a mixed-mode Newton iteration is outlined. A method for the exact computation of the Jacobian matrix, including the exact derivatives with respect to the unknown fundamental frequencies, is presented. Finally, the excellent numerical performance of the new simulation tool is demonstrated by the analysis of a DR-tuned self-oscillating mixer.

INTRODUCTION

Most of the available numerical approaches to the analysis of autonomous nonlinear microwave circuits are restricted to the time-periodic oscillator case [1-11]. This limitation is very reductive, since some of the most interesting and practically important applications of nonlinear circuits involve multitone autonomous operation, typical examples being represented by modulators and self-oscillating mixers. The paper introduces a harmonic-balance approach to the treatment of this class of problems, which is fully compatible with the needs of a general-purpose microwave CAD environment.

Our new method effectively overcomes the limitations of the most significant solutions of the same problem previously proposed in the technical literature. For instance, the Volterra-series technique of [12] is only applicable to weakly nonlinear circuits and requires cumbersome calculations for the evaluation of the Volterra kernels. The substitution method of [13] is applicable to circuits consisting of one-port resistive nonlinearities connected by a linear subnetwork, which is not well suited for microwave CAD applications. In this paper we use a rigorous and absolutely general piecewise harmonic-balance formulation, whereby the problem is reduced to a well-posed nonlinear algebraic system which is solved by a mixed-mode Newton iteration [11]. This allows the simultaneous determination of the steady-state harmonics and of the fundamental frequencies of the quasi-periodic regime. Unrivaled speed and robustness are guaranteed by a general algorithm for the exact computation of the Jacobian matrix.

THE AUTONOMY CONCEPT FOR QUASI-PERIODIC REGIMES

Let us consider a nonlinear microwave circuit operating in a quasi-periodic electrical regime generated by the intermodulation of F sinusoidal tones of incommensurable fundamental angular frequencies ω_i. Any signal a(t) supported by the circuit may be represented in the form

$$a(t) = \sum_{\mathbf{k} \in S} A_{\mathbf{k}} \exp(j\Omega_{\mathbf{k}} t) = \sum_{\mathbf{k} \in S} A_{\mathbf{k}} \exp\left(j \sum_{i=1}^{F} k_i \omega_i t\right) \quad (1)$$

where $\Omega_{\mathbf{k}}$ is a generic intermodulation (IM) product of the fundamentals, i.e.,

$$\Omega_{\mathbf{k}} = \sum_{i=1}^{F} k_i \omega_i = \mathbf{k}^T \omega \quad (2)$$

In (1), (2) k_i is an integer harmonic number, **k** is an F-vector of harmonic numbers, and ω is the F-vector of the fundamentals. The vector **k** in (2) spans a finite subset S of the **k**-space (containing the origin) which will be conventionally named the *signal spectrum*. The Fourier coefficient $A_{\mathbf{k}}$ will be named the *harmonic* of a(t) at $\Omega_{\mathbf{k}}$ (or the **k**-th harmonic of a(t)). Since we want to deal with real signals, **S** must be symmetrical with respect to the origin, and $A_{-\mathbf{k}} = A^*_{\mathbf{k}}$. We shall also denote by S^+ the subset of S such that $\Omega_{\mathbf{k}} \geq 0$ for $\mathbf{k} \in S^+$.

In general, the circuit will be excited by DC (bias) sources for which $\mathbf{k} = \mathbf{0}$, and by a number of free sinusoidal generators of frequencies $\Omega_{\mathbf{g}^{(h)}}$ $(1 \leq h \leq N)$ where $\mathbf{g}^{(h)} \in S^+$. Let us consider the *excitation matrix*

$$\mathbf{G} \equiv \left[\mathbf{g}^{(1)}\ \mathbf{g}^{(2)}\ \ldots\ \mathbf{g}^{(N)}\right] \quad (3)$$

of dimensions F x N. We define the *rank* of G, namely R, as the maximum size of the nonsingular square submatrices of **G** $(R \leq F)$. Then we introduce the following definitions:
1) If R = F the circuit is *non-autonomous.*
2) If R < F the circuit is *autonomous of order M = F - R.*

The physical explanation of these definitions is as follows. First of all, if the circuit is autonomous of order M, we can always define a new set of fundamentals which are linear combinations (with rational coefficients) of the original ones, in such a way that M rows, say the last M ones, of the excitation matrix (3) become zero. We shall assume henceforward that the ω_i's have been selected in this way. In such conditions, the last M of the ω_i's will be referred to as the *free fundamentals.*

In order to find the harmonics of the nonlinear subnetwork response to a multitone excitation of the form (1), we shall make use of the Multiple Fast Fourier Transform (MFFT) technique [14]. According to this method, each of the quantities $z_i = \omega_i t$ appearing in (1) is considered as an independent time variable. From (1) the generic signal a(t) takes the form

$$a(t) = \sum_{\mathbf{k} \in S} A_{\mathbf{k}} \exp\left(j \sum_{i=1}^{F} k_i z_i\right) \quad (4)$$

and is thus viewed as a function defined on an F-dimensional time space $\mathbf{Z}$, which is 2π-periodic in each dimension z_i. The meaning of an autonomy of order M is now clear: the circuit has no forcing term, and thus no phase reference, in M dimensions (out of F) of the time space. Thus the phase of the electrical regime with respect to z_i is indeterminate for $R + 1 \leq i \leq F$. Each arbitrary choice of the phase in the i-th dimension corresponds to one of an ∞^M family of equivalent electrical regimes having the same amplitude spectrum, as shown by (4). For F = 1 this implies the well-known invariance of the time-domain waveforms with respect to a shift of the time origin.

NONLINEAR ANALYSIS BY THE MIXED-MODE NEWTON ITERATION

Let the nonlinear subnetwork be described by the generalized parametric equations [14]

$$\mathbf{v}(t) = \mathbf{u}\left[\mathbf{x}(t)\ ,\ \frac{d\mathbf{x}}{dt}\ ,\ \ldots,\ \frac{d^n\mathbf{x}}{dt^n}\ ,\ \mathbf{x}_D(t)\right]$$
$$\mathbf{i}(t) = \mathbf{w}\left[\mathbf{x}(t)\ ,\ \frac{d\mathbf{x}}{dt}\ ,\ \ldots,\ \frac{d^n\mathbf{x}}{dt^n}\ ,\ \mathbf{x}_D(t)\right] \qquad (5)$$

where $\mathbf{v}(t)$, $\mathbf{i}(t)$ are vectors of voltages and currents at the common ports, $\mathbf{x}(t)$ is a vector of state variables (SV) and $\mathbf{x}_D(t)$ a vector of time-delayed state variables (i.e., $x_{Di}(t) = x_i(t - \tau_i)$). The linear subnetwork has the frequency-domain representation

$$\mathbf{Y}(\omega)\ \mathbf{V}(\omega) + \mathbf{N}(\omega) + \mathbf{I}(\omega) = 0 \qquad (6)$$

where $\mathbf{V}(\omega)$, $\mathbf{I}(\omega)$ are vectors of voltage and current phasors, $\mathbf{Y}(\omega)$ is the linear subnetwork admittance matrix, and $\mathbf{N}(\omega)$ is a vector of Norton equivalent current sources. The set of complex harmonic-balance errors at a generic IM product $\mathbf{\Omega_k}$ ($\mathbf{k} \in \mathbf{S}^+$) has the expression

$$\mathbf{E}_k = \mathbf{Y}(\Omega_k)\ \mathbf{U}_k + \mathbf{N}(\Omega_k) + \mathbf{W}_k \qquad (7)$$

where $\mathbf{U}_k$, $\mathbf{W}_k$ are vectors of harmonics of $\mathbf{u}(t)$, $\mathbf{w}(t)$. All vectors in eq. (5)-(7) have a same size n_d equal to the number of common ports. Similarly, the vector of SV harmonics at Ω_k will be denoted by $\mathbf{X}_k$. In order to avoid the use of negative frequencies, the nonlinear solving system is formulated in terms of vectors $\mathbf{E}$, $\mathbf{X}$ of real and imaginary parts of the errors (7) and of the state-variable harmonics, respectively. The size of these vectors is $N_T = n_d(2n_H + 1)$, where n_H is the number of IM products to be considered (not including DC). We shall assume that the circuit makes available M independent *tuning parameters*, which usually represent bias voltages and/or free parameters of the linear subnetwork. The solving system is thus written in the form

$$\mathbf{E}(\mathbf{X}, \mathbf{P}, \mathbf{V}_\omega) = 0 \qquad (8)$$

where $\mathbf{P}$, $\mathbf{V}_\omega$ are the M-vectors of the tuning parameters and of the free fundamentals, respectively. If all these are treated as quantities to be found, (8) is a nonlinear system of N_T equations in $N_T + 2M$ real unknowns. However, as it was shown in the previous section, the phases of the electrical regime with respect to the free fundamentals are not determined by (8), and can thus be arbitrarily selected. This is usually accomplished by setting to zero the imaginary parts of the SV harmonics at M intermodulation products representing linearly independent combinations of the free fundamentals (*reference harmonics*). Thus the vector $\mathbf{X}$ contains only N_T - M real quantities to be determined, and (8) is in reality a system of N_T equations in N_T + M unknowns.

Two kinds of situations are most often encountered in practice: i), $\mathbf{V}_\omega$ is fixed and $\mathbf{X}$, $\mathbf{P}$ must be determined (*tuning problem*), and, ii), $\mathbf{P}$ is fixed and $\mathbf{X}$, $\mathbf{V}_\omega$ must be determined (*analysis problem*). Both cases result in well-posed nonlinear systems of N_T real equations in as many unknowns. Each system can be solved by a plain Newton iteration, with a set of unknowns simultaneously including the reduced vector of SV harmonics, and the tuning parameters or the free fundamentals. Because of this hybrid vector of unknowns, this will be referred to as a *mixed-mode* Newton iteration as opposed to the purely harmonic Newton used for non-autonomous circuits [11]. Of course, mixed problems where M elements of $\mathbf{P} \cup \mathbf{V}_\omega$ are fixed and the remaining M must be found may also occur, and can be solved in a similar way.

For an efficient implementation of the Newton algorithm it is of paramount importance that the Jacobian matrix of the HB errors with respect to the unknowns be computed exactly, rather than by numerical perturbations [15]. This simultaneously reduces the Jacobian evaluation cost and the number of Newton iterations required to solve (8), due to the increased accuracy of the derivatives. From (7) we get ($\mathbf{k}$,$\mathbf{s} \in \mathbf{S}^+$)

$$\frac{\partial \mathbf{E}_k}{\partial f[\mathbf{X}_s]} = \mathbf{Y}(\Omega_k)\frac{\partial \mathbf{U}_k}{\partial f[\mathbf{X}_s]} + \frac{\partial \mathbf{W}_k}{\partial f[\mathbf{X}_s]}$$
$$\frac{\partial \mathbf{E}_k}{\partial \mathbf{P}} = \frac{\partial \mathbf{Y}}{\partial \mathbf{P}}(\Omega_k)\ \mathbf{U}_k + \frac{\partial \mathbf{N}}{\partial \mathbf{P}}(\Omega_k) \qquad (9)$$
$$\frac{\partial \mathbf{E}_k}{\partial \omega_i} = k_i\left[\frac{\partial \mathbf{Y}}{\partial \omega}(\Omega_k)\ \mathbf{U}_k + \frac{\partial \mathbf{N}}{\partial \omega}(\Omega_k)\right] + \mathbf{Y}(\Omega_k)\frac{\partial \mathbf{U}_k}{\partial \omega_i} + \frac{\partial \mathbf{W}_k}{\partial \omega_i}$$

where f[•] may denote either the real or the imaginary part.

The derivatives of $\mathbf{Y}$ and $\mathbf{N}$ with respect to both circuit parameters and frequency can be computed by adjoint-network techniques [16]. The derivatives of the voltage and currents harmonics $\mathbf{U}_k$, $\mathbf{W}_k$ are found in the following way. For the voltages (e.g.) we first introduce the Fourier expansions

$$\frac{\partial u}{\partial y_m} = \sum_{p \in S_d} C_{m,p}\ \exp(j\Omega_p t)$$
$$\frac{\partial u}{\partial x_D} = \sum_{p \in S_d} C_p^D\ \exp(j\Omega_p t) \qquad (10)$$

where $y_0 = \mathbf{x}$, $y_m = d^m\mathbf{x}/dt^m$ ($1 \leq m \leq n$), and the *derivatives spectrum* $\mathbf{S}_d$ is usually larger than the signal spectrum. Then for the derivatives of $\mathbf{U}_k$ with respect to the SV harmonics we use the expressions given in refs. [14], [15]. For the derivatives with respect to the free fundamentals we have

$$\frac{\partial \mathbf{U}_s}{\partial \omega_i} = j\sum_{m=1}^{n} m\left[\sum_k k_i\,(j\Omega_k)^{m-1}\,C_{m,s-k}\,\mathbf{X}_k\right] - j\left[\sum_k k_i\,C^D_{s-k}\tau\,\exp(-j\Omega_k\tau)\,\mathbf{X}_k\right] \qquad (11)$$

where τ is the diagonal matrix of the time delays.

The derivatives of the current harmonics are computed in a similar way. The expression (11) is believed to be new, and represents an important mathematical result of the present work.

The computation of the derivatives of **Y** usually represents a major contribution to the overall analysis cost. In particular, finding $\partial \mathbf{Y}/\partial \omega$ is more expensive than finding $\partial \mathbf{Y}/\partial \mathbf{P}$, since the former involves all the reactive circuit components, and the latter only those belonging to the tuning parameters. Thus, in general tuning is a faster process than analysis. Much more so if all the tuning parameters are bias voltages, since in this case the second of (9) reduces to $\delta_k^0\, \partial \mathbf{N}/\partial \mathbf{P}(0)$. The admittance then remains constant throughout the iteration and its derivatives are not required. These facts can be effectively exploited in many important applications, such as the computation of the frequency-voltage characteristic of VCO's [11].

The pattern of possible solutions of the system (8) may be estremely complicated in the multidimensional case. A general solution would require a global stability analysis of the parametrized circuit, based on an extended concept of Hopf bifurcation [17, 18]. This kind of analysis is beyond the scope of the present paper and will be discussed in a future work. For the time being, we shall limit ourselves to the assumption that the quasi-periodic regime of interest is a perturbation (not necessarily small) of an *a priori* known (and usually simpler) steady state. This class of problems includes a wide variety of cases of practical interest, such as self-oscillating mixers, modulators, phase-locked oscillators, and so forth.

All these applications can be treated by the mixed-mode Newton iteration (possibly coupled with a continuation method) in a most efficient way. If necessary, a suitable initial point for the Newton algorithm can always be found in a straightforward way by carrying out a preliminary iteration (e.g., by a quasi-Newton method) with the reference harmonics held fixed, as it is discussed in [18]. Note that in many cases (such as in the example discussed below) the initial-point problem does not exist, since the multitone analysis can be safely started from the results of a free-running oscillator optimization [19].

AN EXAMPLE OF APPLICATION

Let us consider the self-oscillating mixer whose schematic topology is given in fig. 1. The circuit consists of a reflection-type DRO using a 150 μm FET as the active device. The required capacitive source feedback and inductive gate feedback are realized by an open stub and a loaded microstrip line coupled to a dielectric resonator, respectively. The same line is also used to input the RF signal. At the output a lumped IF filter/matching circuit is fed through a microstrip line used to provide the optimum load reactance to the FET drain at the LO frequency.

The IF section lying beyond point L in fig. 1 is first suppressed, and the local oscillator is optimized for maximum drain current at a fundamental frequency of 8 GHz by the technique described in [20], with 4 harmonics taken into account. The IF matching circuit is then introduced and is designed in such a way as to load the FET drain by a real impedance of about 150 Ω at the required intermediate frequency of 510 MHz. The circuit is then analyzed by the method discussed in the previous section (autonomy of order 1, time-periodic regime), and the drain voltage spectrum shown in fig. 2 a) is obtained. The observed frequency shift of about -20 kHz with respect to the nominal design value of 8 GHz is due to the small additional reactive loading of the FET drain introduced by the IF filter. An RF signal having an available power of -6 dBm at 8.51 GHz is then injected into the gate through the input microstrip, and the circuit is analyzed once again (autonomy of order 1, two-dimensional quasi-periodic regime). The resulting drain voltage spectrum is shown in fig. 2 b). The additional LO frequency shift due to the injection of the RF signal is about -166 kHz. The transducer conversion gain of the self-oscillating mixer is about -3 dB without RF input matching. No starting-point problems are observed for these analyses.

Because of the superlinear rate of convergence of the Newton iteration in the vicinity of the solution, finding the frequency of oscillation with very high accuracy requires a limited numerical effort. In both situations referred to in fig. 2, the unknown frequency is identified with a relative error smaller than 10^{-9}; the number of required Newton iterations is 7 and 12, respectively. This allows an excellent numerical control of the small frequency shifts that are typical of dielectric-resonator tuned oscillators and self-oscillating mixers. On a SUN SPARCstation 2, the analysis of fig. 2 a) takes about 0.62 CPU seconds (not including I/O), 61% of which are spent in the linear subnetwork analysis. In turn, the multitone analysis of fig. 2 b) takes about 3.9 seconds, 33% of which are spent in the linear subnetwork analysis. It is thus clear that the nonlinear analysis algorithm is extremely efficient. For comparison, a time-domain analysis would be at least two orders of magnitude slower [21] due to the high Q of the dielectric resonator (around 2700 in the present case). Also, the MFFT approach [14] completely eliminates the leakage error produced by the application of ordinary Fourier transformation to non-periodic waveforms [21].

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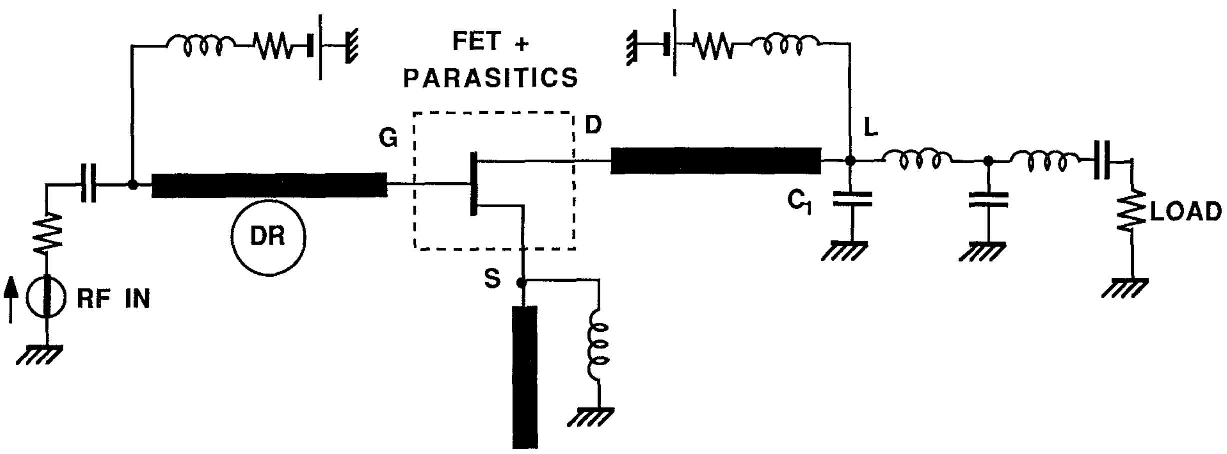

Fig. 1 - Schematic topology of a dielectric-resonator tuned self-oscillating mixer.

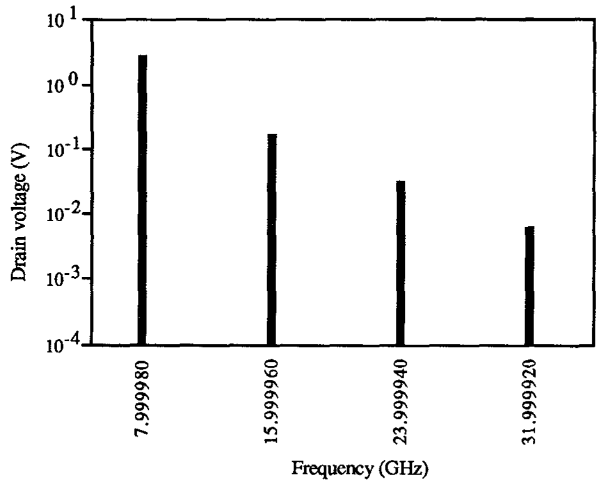

Fig. 2 a) - Drain voltage spectrum of the local oscillator in the absence of RF signal.

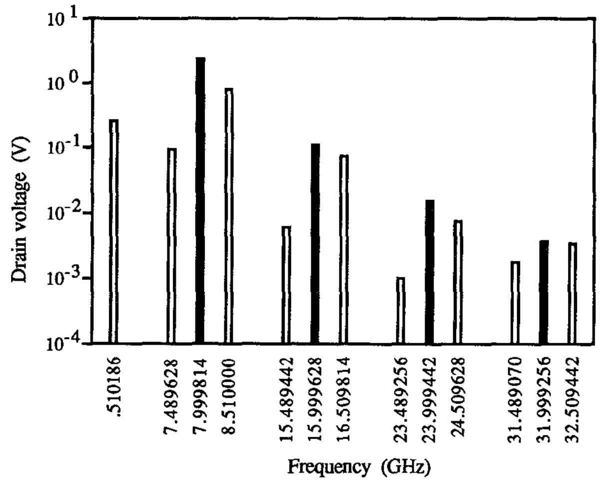

Fig. 2 b) - Drain voltage spectrum of the self-oscillating mixer for an available RF input power of -6 dBm.